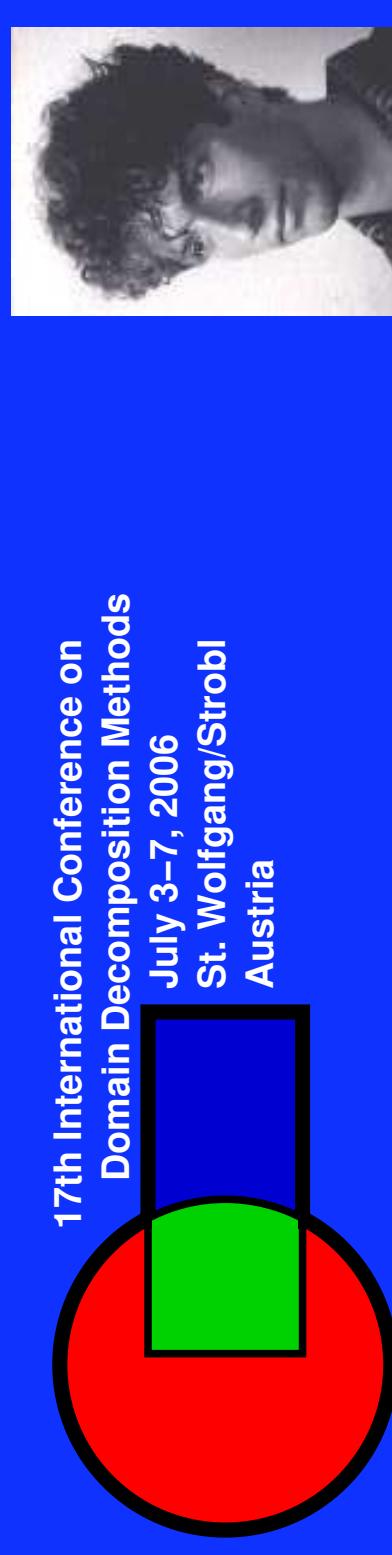


Domain decomposition on different levels of the Jacobi-Davidson method

abstract

Most computational work of Jacobi-Davidson [3] is an iterative method suitable for computing solutions of large eigenvalue problems, due to a so-called correction equation on the intermediate level. In [2, 11] a strategy for the computation of (approximate) solutions of this correction equation was proposed. The strategy is based on a nonoverlapping additive Schwarz method with locally optimized coupling parameters [5, 4] in order to reduce the wall clock time and local memory requirements.

The poster presented here discusses the aspect that the domain decomposition approach can also apply on the highest level of the Jacobi-Davidson method. Numerical experiments show that for large-scale eigenvalue problems this aspect is beneficial. Furthermore, the computation of the domain decomposition approach and the enhanced iterative method for eigenvalue problems, that is, an elegant edition between parallel performance on distributed systems and absorbing boundary conditions at the interfaces between the subdomains.



references

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correction equation

The correction equation describes an (inexact) Newton step. This special linear system has equal dimensions as the original eigenvalue problem. For not too low dimensional problems, however, Jacobi-Davidson is feasible and the correction equation doesn't need to be solved in this ingredient. Conventional work of the Jacobi-Davidson method for the standard and the QZ-algorithm for generalized eigenvalue problems (like the QR-algorithm for the standard and the QZ-algorithm for the generalized eigenvalue problem) can not cope well with such high dimensions. In such a situation one may add a subspace method. Among these subspace methods the Jacobi-Davidson method [3] offers many advantages and flexibility, amongst others good preconditioners as domain decomposition can be fully exploited.

Eigenvalue problems show up in a diversity of scientific disciplines. In most situations the object of study is described by some partial differential equation. High resolution discretizations lead to (generalized) eigenvalue problems of very high dimension. Conventional dense methods (like the QR-algorithm for the standard and the QZ-algorithm for the generalized eigenvalue problem) can not cope well with such high dimensions. In such a situation one may add a subspace method. Among these subspace methods the Jacobi-Davidson method [3] offers many advantages and flexibility, amongst others good preconditioners as domain decomposition can be fully exploited.

For the computation of a solution (λ, \mathbf{x}) to the standard eigenvalue problem

$$\mathbf{A} \mathbf{x} = \lambda \mathbf{x}$$

in each iteration step Jacobi-Davidson:

- constructs $H \equiv V \mathbf{A} V'$,
- solves $H s = s$, and
- computes $\mathbf{u} = \theta s$, and

where the columns of V form an orthonormal basis for the search subspace,

corrects the approximate eigenvector \mathbf{u} by computing a correction vector \mathbf{t} from the correction equation:

$$\mathbf{P} \equiv \mathbf{I} - \frac{\mathbf{u} \mathbf{u}^T}{\mathbf{u}^T \mathbf{u}}, \quad \mathbf{B} \equiv \mathbf{A} - \theta \mathbf{I}, \quad \mathbf{r} \equiv \mathbf{B} \mathbf{u},$$

expands the search subspace with the orthogonal component $(\mathbf{I} - \mathbf{V} \mathbf{V}^T) \mathbf{t}$ of the correction extracts an approximate solution (θ, \mathbf{u}) from a search subspace by

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